Stackelberg 2 firms linear demand

Problem Statement

Consider a Stackelberg duopoly where two firms compete in quantity in a market with linear demand. Firm 1 is the leader and firm 2 is the follower. The market demand function is given by:

$$P = a - bQ, \tag{1}$$

where P is the price, $Q = q_1 + q_2$ is the total quantity in the market, q_1 and q_2 are the quantities produced by firm 1 and 2, respectively, and a and b are positive constants. Both firms have different but constant marginal costs, denoted by c_1 and c_2. Find the equilibrium quantities and price for both firms.

Solution

Profit function

The profit function for each firm is defined as revenue minus cost, as a function of the quantity produced. For firm 1:

$$\pi_1(q_1, q_2) = Pq_1 - c_1q_1 = (a - b(q_1 + q_2))q_1 - c_1q_1. \tag{2}$$

For firm 2:

$$\pi_2(q_1, q_2) = Pq_2 - c_2q_2 = (a - b(q_1 + q_2))q_2 - c_2q_2. \tag{3}$$

Reaction function

To find the reaction function for firm 2, we first calculate the first-order condition for maximizing profit with respect to q_2:

$$\frac{\partial \pi_2(q_1, q_2)}{\partial q_2} = a - 2bq_2 - bq_1 - c_2. \tag{4}$$

Setting the first-order condition to zero and solving for q_2:

$$q_2^*(q_1) = \frac{a - bq_1 - c_2}{2b}. \tag{5}$$

Stackelberg leader's optimization

Now, we substitute the reaction function of the follower into the leader's profit function:

$$\pi_1(q_1, q_2^*(q_1)) = (a - b(q_1 + \frac{a - bq_1 - c_2}{2b}))q_1 - c_1q_1. \tag{6}$$

Next, we find the first-order condition for maximizing the leader's profit with respect to q_1:

$$\frac{\partial \pi_1(q_1, q_2^*(q_1))}{\partial q_1} = a/2 - bq_1 + c_2/2 - c_1. \tag{7}$$

Setting the first-order condition to zero and solving for q_1:

$$q_1^* = \frac{a + c_2 - 2c_1}{2b}. \tag{8}$$

Equilibrium quantities and price

To find the equilibrium quantity for firm 2, we substitute the optimal quantity for firm 1 into firm 2's reaction function:

$$q_2^* = \frac{a - bq_1^* - c_2}{2b} = \frac{a - b\left(\frac{a + c_2 - 2c_1}{2b}\right) - c_2}{2b}. \tag{9}$$

Simplifying, we get:

$$q_2^* = \frac{a + 2c_1 - 3c_2}{4b}. \tag{10}$$

Now, we can calculate the equilibrium price using the market demand function and the equilibrium quantities:

$$P = a - b(q_1^* + q_2^*) = a - b\left(\frac{a + c_2 - 2c_1}{2b} + \frac{a + 2c_1 - 3c_2}{4b}\right). \tag{11}$$

$$P = a - b(q_1^* + q_2^*) = a - b\left(\frac{3a - c_2 - 2c_1}{4b}\right). \tag{12}$$

Simplifying, we get:

$$P = \frac{a + c_2 + 2c_1}{4}. \tag{13}$$